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# Anomalies and Fayet-Iliopoulos terms on warped orbifolds and large hierarchies

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Abstract: The anomalies in five-dimensional orbifold theories are examined in a generic type of non-factorizable geometries. In spite of complicated fermion wavefunctions, the shape of anomaly is found to be identical to that of flat theories. In particular it is split evenly on the orbifold fixed points. This result also follows from the arguments on the AdS/CFT correspondence and an anomaly cancellation mechanism. The cancellation with Chern-Simons term works if the four-dimensional effective theory is free from chiral anomalies. We also discuss the Fayet-Iliopoulos (FI) term in warped supersymmetric theories. Unlike the gauge anomaly, radiatively-generated FI terms reside not only on the orbifold fixed points but also in the whole five-dimensional bulk. This bulk FI term is necessary to have unbroken supersymmetry. The effect of FI term is to generate supersymmetric masses for charged hypermultiplets, which are no longer constant but have metric factor dependence. We calculate the spectrum and wavefunctions of Kaluza-Klein modes in the presence of the FI term and discuss phenomenological implications to quark-lepton masses and large scale hierarchies.

Keywords: Field Theories in Higher Dimensions, Anomalies in Field and String Theories, Supersymmetric Standard Model.

## Contents



## 1. Introduction

In these years, much progress has been made with theories in higher dimensions. The geometries of extra dimensional spacetime have been playing an essential role for explaining various unresolved problems in particle physics, e.g. the hierarchy problem. Among them, a model which realizes a large mass hierarchy is presented [1] by use of a non-factorizable form of five dimensional metric.

In the model of ref. [1], all the standard model fields are confined on a brane. The possibilities that some of matter and gauge fields propagate in the bulk space has also been pursued in [2]. There orbifold projections are imposed on the fields at the boundaries in order to realize chiral spectrum in effective four-dimensional theories. An orbifold theory is often chiral and may become an anomalous gauge theory. To be a consistent theory, such anomaly should be canceled. Although a higher-dimensional field theory is apparently not renormalizable and a cutoff scale is not so high compared with the compactification scale, one can discuss quantities of higher-dimensional theories, such as anomalies and renormalization of couplings up to the cutoff scale. The gauge anomaly in an orbifold field theory has been calculated for the flat extra dimensions and found that the anomaly is localized on the orbifold fixed points [3, 4]. Even when the low-energy effective theory has vector-like mass spectrum, gauge anomalies are induced by one-loop fermion diagrams in higher dimensions. In this case, however, the total anomaly is canceled out by adding a Chern-Simons term to the action [5, 6]. Keeping in mind phenomenological interests



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of non-factorizable geometries, in the first half of this paper we study gauge anomalies of orbifold theories in a generic type of non-factorizable geometries (including the metric of [1]). We will find that the anomaly is localized in the exactly same way as in flat theories and is canceled if the effective four-dimensional theory is vector like. It might be difficult to understand this evenly-split anomaly because wavefunctions of fermions are highly complicated depending on five-dimensional metrics. We will show that the anomaly should be split equally on the fixed points from the theoretical arguments on the AdS/CFT correspondence and an anomaly cancellation mechanism.

When one considers a supersymmetric U(1) gauge theory, the Fayet-Iliopoulos (FI) term is important for studying the vacuum and low-energy spectrum of the system. The appearance of FI term is deeply connected to gravitational anomalies in supergravity theories. For flat supersymmetric orbifold theories, it is known that quantum effects induce non-vanishing FI terms on the fixed points [7, 4] and hence a condition for supersymmetric vacuum is modified [8, 6]. In the second half of this paper, we investigate the structure of FI term in supersymmetric five-dimensional theory on the warped background. Unlike the flat background, we find that FI terms are induced not only on the fixed points but also in the whole five-dimensional bulk. When the effective theory is free from gravitational anomalies, the five-dimensional theory has a supersymmetric vacuum even in the presence of the bulk and boundary FI terms. In this vacuum, bulk masses for charged hypermultiplets are generated which depend on the warped metric factor and are proportional to the coefficients of FI terms. We discuss the Kaluza-Klein (KK) mass spectrum in the presence of such bulk mass terms and study characteristic behavior of wavefunction profiles. In particular, the localization of light modes in the extra dimension is important for four-dimensional phenomenology. As typical examples, several realistic models are constructed in this framework to realize Yukawa and Planck/weak mass hierarchies.

## 2. Gauge anomaly in curved spacetime

For later comparison, we first review the gauge anomaly in flat five-dimensional orbifold theories. The fifth dimension parameterized by $y$ is compactified on the $S^1/\mathbb{Z}_2$ orbifold with two fixed points $y = 0$ and $\pi R$. The appropriate boundary conditions at the fixed points are introduced for a bulk gauge field $A_M(x, y)$ and a Dirac spinor $\Psi(x, y)$. They are defined so as to be consistent with the $\mathbb{Z}_2$ orbifold:

$$\begin{aligned}
A_\mu(x,y) &= +A_\mu(x,-y)\,, & A_\mu(x,y-\pi R) &= +A_\mu(x,-(y-\pi R))\,, \\
A_5(x,y) &= -A_5(x,-y)\,, & A_5(x,y-\pi R) &= -A_5(x,-(y-\pi R))\,, \\
\Psi(x,y) &= +\gamma_5\Psi(x,-y)\,, & \Psi(x,y-\pi R) &= +\gamma_5\Psi(x,-(y-\pi R))\,. 
\end{aligned} \tag{2.1}$$

The indices of Roman letters $M$, $N, \ldots,$ run over all dimensions and the Greek letter indices $\mu$, $\nu, \ldots,$ run only over the first four dimensions. The $4 \times 4$ matrix $\gamma_5$ is diag $(-1, -1, +1, +1)$. With these boundary conditions, $A_\mu$ and the right-handed component $(\frac{1+\gamma_5}{2})\Psi$ have four-dimensional massless modes. Due to the fermion boundary conditions, the theory is chiral on the fixed points and the gauge anomaly might be present in the divergence of gauge current $J^M = \bar{\Psi}\Gamma^M\Psi$. Since the anomaly is a low-energy effect, its form can be calculated via Kaluza-Klein reduction procedure and is found [3]

$$\partial_M J^M = \frac{g^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\left[\delta(y)+\delta(y-\pi R)\right]. \tag{2.2}$$

The delta function is defined as $\int_0^{\pi R} f(y)\delta(y)dy = f(0)/2$. The anomaly coefficient $g^2/32\pi^2$ is proportional to the chiral anomaly for a four-dimensional Weyl fermion. In the effective theory viewpoint, the anomaly (2.2) comes from the zero-mode contribution and a Chern-Simons term which is generated by integration of heavy KK fermions.

Several interesting properties are in the form of gauge anomaly (2.2); (i) there is no anomaly in the bulk. It is localized at the fixed points as expected from the chiral boundary conditions, (ii) the anomaly is equally split between the two fixed points, and (iii) the integration of the anomaly over $y$ direction is nonzero which is understood from the fact that there is a chiral zero mode. For different boundary conditions of fermions that do not leave any chiral zero modes, the integrated anomalies in effective theories vanish [4, 6]. Various other aspects of localized anomalies in flat spacetime have been studied [9].

Now let us consider five-dimensional gauge theories in curved spacetime and present the possible form of one-loop gauge anomalies. Throughout this paper, the gravity is treated as a background as we are interested in consistency of field theory. We particularly focus on the following non-factorizable background metric:

$$ds^2 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2\,, \tag{2.3}$$

where $\eta_{\mu\nu}$ is the four-dimensional flat Minkowski metric, and we normalize the factor $a(y)$ so that $a(0)=1$. A phenomenologically interesting example is the anti de-Sitter (AdS) geometry where $a(y)$ is given by the warp factor $e^{-k|y|}$ ($k$ is the AdS curvature). In this section, however, we do not specify the metric factor, and will see that the gauge anomaly on the background (2.3) does not depend on the form of $a(y)$.

We consider the action for a gauge field $A_M(x,y)$ and a Dirac spinor $\Psi(x,y)$ on the background (2.3)

$$S = \int d^4x dy\sqrt{-g}\left[-\frac{1}{4}F_{MN}F^{MN} + \bar{\Psi}i\Gamma^M D_M\Psi - m(y)\bar{\Psi}\Psi\right], \tag{2.4}$$

where $\Gamma_M$ are the gamma matrices in five dimensions and $D_M$ is the covariant derivative including the spin connection and the gauge field; $D_M = \partial_M - igA_M + \frac{1}{8}\omega_M^{PQ}[\Gamma_P,\Gamma_Q]$. The bulk mass parameter $m(y)$ must obey the condition $m(-y)=-m(y)$ consistent with the $\mathbb{Z}_2$ orbifolding. We will consider the Abelian example and the extension to non-Abelian gauge theory is straightforward. The action is classically invariant under the gauge transformation with a gauge parameter $\Lambda(x,y)$

$$\begin{aligned} A'_M(x,y) &= A_M(x,y) + \partial_M\Lambda(x,y)\,, \\ \Psi'(x,y) &= e^{i\Lambda(x,y)}\Psi(x,y)\,. \end{aligned} \tag{2.5}$$

The corresponding conserved current ($\eta^{MN}\partial_M J_M = 0$) is

$$J_M = \sqrt{-g}\bar{\Psi}\Gamma_M\Psi = \begin{cases} a^3(y)\bar{\Psi}\gamma_\mu\Psi & M=\mu \\ a^4(y)\bar{\Psi}(i\gamma_5)\Psi & M=5\,, \end{cases} \tag{2.6}$$

where $\gamma_\mu$ are the gamma matrices in four dimensions.

As in the flat case, we compactify the fifth dimension on a line segment $y = [0, \pi R]$ and introduce a brane at each boundary.[1] The orbifold boundary conditions for bulk fields at $y = 0$ and $y = \pi R$ are also similar to the flat case. In particular, the Dirac fermion is subject to the following parity conditions:

$$\Psi(x,y) = \gamma_5 \Psi(x,-y)\,, \qquad \Psi(x,y-\pi R) = \eta\gamma_5\Psi(x,-(y-\pi R))\,. \tag{2.7}$$

For the conditions consistent with the $\mathbb{Z}_2$ orbifold, $\eta$ has to satisfy $\eta^2 = 1$. For $\eta = +1$, we have a massless right-handed chiral fermion in the low-energy effective theory. On the other hand, for $\eta = -1$, all KK fermions become massive, and massless modes are projected out. A five-dimensional gauge theory is expected to have no chiral anomaly because of its vector-like nature. However, with the chiral boundary conditions (2.7), the gauge current (2.6) suffers from chiral anomalies at quantum level. In particular, although the effective theory is vector-like for the $\eta = -1$ case, we will see the anomalies are induced onto the boundaries.

Now we calculate the chiral anomaly on the curved background metric (2.3) via the KK point of view. We decompose $\Psi$ into the four-dimensional KK modes

$$\Psi(x,y) = \sum \psi_n^+(x)\chi_n^+(y) + \psi_n^-(x)\chi_n^-(y)\,, \tag{2.8}$$

where $\psi_n^{+(-)}(x)$ are four-dimensional right-(left-) handed fermions. The equations of motion are given by

$$\begin{aligned}\left[\partial_y + 2\frac{\partial_y a}{a} - m(y)\right]\chi_n^+ &= +\frac{m_n}{a}\chi_n^-\,,\\ \left[\partial_y + 2\frac{\partial_y a}{a} + m(y)\right]\chi_n^- &= -\frac{m_n}{a}\chi_n^+\,,\end{aligned} \tag{2.9}$$

where $m_n$ are the four-dimensional masses ($p_\mu^2 = -m_n^2$). In what follows, we consider a massless Dirac fermion $m(y) = 0$ for an illustrative example. However, it will be found that the shape of gauge anomalies is insensitive to a value of $m(y)$ and therefore the analysis with $m(y) = 0$ gives generic results. The mass eigenmodes for $m(y) = 0$ are

$$\chi_n^+(y) = \sqrt{\frac{2}{A(\pi R)}}\, a^{-2}(y)\cos\left[m_n A(y)\right], \tag{2.10}$$

$$\chi_n^-(y) = \sqrt{\frac{2}{A(\pi R)}}\, a^{-2}(y)\sin\left[m_n A(y)\right], \tag{2.11}$$

and $A(y) \equiv \int_0^y dy' a^{-1}(y')$. These eigenfunctions satisfy the orbifold conditions at the $y = 0$ boundary. The normalization factors are determined so that the four-dimensional modes have canonical kinetic terms $\int_0^{\pi R} dy\, a^3(y)\chi_m^\pm(y)\chi_n^\pm(y) = \delta_{mn}$. The mass eigenvalues $m_n$

[1] The finite four-dimensional Newton constant can be realized with infinite extra dimension(s) depending on the factor $a(y)$. Note, however, that the zero modes of gauge fields have constant wavefunctions along $y$ direction and the normalizability must require a finite size of extra dimension (two boundaries) in the absence of some localization mechanism for the zero modes.

are fixed by the boundary condition at $y = \pi R$,

$$m_n = \begin{cases} \dfrac{n\pi}{A(\pi R)} & (\eta = +1) \\ \dfrac{(n+1/2)\pi}{A(\pi R)} & (\eta = -1) \end{cases} \qquad n = 0, 1, 2, \ldots . \tag{2.12}$$

Along the line of ref. [3], we calculate the five-dimensional gauge anomaly by summing up familiar four-dimensional anomalies of the KK modes. With noting the metric factor appears in the current, the form of anomaly is expressed in terms of the eigenfunctions

$$\eta^{MN}\partial_M J_M = \frac{g^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\, a^3(y)\left(\sum|\chi_n^+|^2 - \sum|\chi_n^-|^2\right). \tag{2.13}$$

Substituting the eigenfunctions (2.10) and (2.11), the summation over all the KK modes becomes

$$\begin{aligned} \sum|\chi_n^+|^2 - \sum|\chi_n^-|^2 &= \frac{2a^{-4}(y)}{A(\pi R)}\left[\sum\cos^2[m_n A(y)] - \sum\sin^2[m_n A(y)]\right] \\ &= \frac{a^{-4}(y)}{A(\pi R)}\left[\delta\Big(\frac{A(y)}{A(\pi R)}\Big) + \eta\delta\Big(\frac{A(y)}{A(\pi R)} - 1\Big)\right] \\ &= \delta(y) + \eta a^{-3}(\pi R)\delta(y - \pi R). \end{aligned} \tag{2.14}$$

We thus obtain the five-dimensional gauge anomaly on the curved metric (2.3):

$$\eta^{MN}\partial_M J_M = \frac{g^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}[\delta(y) + \eta\delta(y - \pi R)]. \tag{2.15}$$

From this form one can see that the anomaly appears only at the boundaries and furthermore the sizes of anomalies on the two boundaries are equal. It is interesting that the result is exactly same as that in the flat spacetime. There is no dependence on the metric factor $a(y)$. For a nonzero Dirac mass $m(y)$, the wavefunctions of fermion KK modes have completely different forms. We have, however, checked that even in that case, the gauge anomaly is not affected by the presence of mass term and therefore is given by (2.15). If there is a chiral four-dimensional fermion living on a brane, it induces a localized anomaly with the coefficient $g^2/16\pi^2$ at $y = 0$ or $y = \pi R$, depending on the position where the fermion lives.

Intuitively, it might be difficult to understand the fact that the anomaly is split evenly between the two boundaries. This is mainly because, on the curved metric, the wavefunctions of KK fermions are highly curved and generally take rather different values on the two boundaries. For example, for a fermion in the $\mathrm{AdS}_5$ geometry with a bulk mass smaller than $k/2$, all KK fermions including the massless mode are strongly peaked at the $y = \pi R$ brane with the exponential warp factor. Nevertheless our result (2.15) shows that the gauge anomaly is not localized in a lopsided way on that brane. In the following, we argue two theoretical grounds why it should be so in a five-dimensional orbifold theory.

The first argument is related to a condition of anomaly cancellation. As mentioned before, for $\eta = -1$ (without boundary fermions), the low-energy effective theory has vector-like mass spectrum and therefore is supposed not to suffer from any anomalies. This implies that the apparent anomaly (2.15) does not introduce quantum breaking of gauge symmetry but, with appropriate regularization, it should be canceled by additional local terms which are allowed by the symmetry. In the present five-dimensional theory, an adequate term for this purpose is the Chern-Simons term:

$$S_{\mathrm{CS}} = \int d^4x dy\, \rho(y) \epsilon^{MNPQR} A_M F_{NP} F_{QR} \,. \tag{2.16}$$

The coefficient $\rho(y)$ is periodic and must be an odd function of $y$ such that it respects the $\mathbb{Z}_2$ symmetry. It is easily found that the gauge variation of $S_{\mathrm{CS}}$ gives rise to the right term to cancel the anomaly. The vanishing anomaly is ensured if the condition

$$\frac{g^2}{32\pi^2} \left[\delta(y) - \delta(y - \pi R)\right] - \partial_y \rho(y) = 0 \tag{2.17}$$

is satisfied. We find the solution which is given by

$$\rho(y) = \frac{g^2}{64\pi^2} \epsilon(y) \,, \tag{2.18}$$

where $\epsilon(y)$ is the sign function with anti-periodicity $\pi R$ and is allowed by the symmetry. It should be noticed that the localized anomaly can be canceled by the Chern-Simons term only if the two coefficients of delta functions in the anomaly are equal to each other and have a relative minus sign ($\eta = -1$). Thus, this argument suggests that the localized anomaly should have the form (2.15) and be independent of the mass parameter $m(y)$. With an appropriate regularization scheme, the Chern-Simons term with the coefficient (2.18) is indeed generated at loop level. It is also clear that the anomaly for $\eta = +1$ cannot be canceled by Chern-Simons terms. This is because massless modes necessarily appear in the low-energy theory and the regularization à la Pauli-Villars does not work.

Another theoretical support comes from the AdS/CFT correspondence [10, 11], which claims that a five-dimensional gravity theory on the AdS$_5$ is holographic dual to a four-dimensional conformal field theory (CFT) in the large $N$ limit. It is also argued that an AdS$_5$ theory with boundaries, e.g. the Randall-Sundrum model [1], has a certain four-dimensional dual description [12, 13]. The analysis of four-dimensional dual theory gives an understanding of the evenly localized anomaly in the AdS$_5$.

In the AdS/CFT correspondence, the fifth dimension is encoded as the energy scale in the four-dimensional CFT. A larger (smaller) value of the fifth dimension $y$ corresponds to the infrared (ultraviolet) in four dimensions. Modifying the AdS side by introducing the Planck brane is interpreted in the CFT as introducing a ultraviolet (UV) cutoff of the Planck scale and turning on the coupling to gravity. The structure of Planck brane theory determines the details of UV deformation of CFT. On the other hand, the presence of the IR brane implies the breaking of conformal symmetry in the IR regime. We are now interested in a bulk gauge theory $G$ on the AdS background, which means that the subgroup $G$ of global symmetry in the CFT is weakly gauged. It is indeed checked that the anomaly in CFT is related to the Chern-Simons term in the AdS side [11].

To examine the anomaly and its cancellation in the CFT side, we need to know how various types of AdS fields are mapped into the four-dimensional description under the duality. It is known that localized four-dimensional fields on the UV brane are interpreted as sources of CFT. They do not have direct couplings to CFT in the four-dimensional gravity point of view, but $G$ gauge interactions connect the two sectors ($G$ is weakly gauged). On the other hand, four-dimensional fields on the IR brane correspond to massless composites of the CFT. This is suggested from the fact that the IR brane fields are strongly coupled to the KK-excited gravitons which are localized on the IR brane. In a similar manner, KK-excited modes of bulk fields correspond to massive bound states of the broken CFT.

Matching of AdS bulk fields is also influenced by boundary conditions at the two branes. There are four types of boundary conditions on fermion fields, $\psi^{(\pm\pm)}$. The first and second indices indicate the boundary conditions at the UV and IR branes, respectively. The $+$ sign denotes a Neumann type and $-$ a Dirichlet one. With the notation (2.7), the $\eta = +1$ case has $\psi^{(++)}$ and $\psi^{(--)}$ fermions, and the $\eta = -1$ case $\psi^{(+-)}$ and $\psi^{(-+)}$. The fields which obey the Neumann boundary conditions at the UV brane, i.e. $\psi^{(++)}$ and $\psi^{(+-)}$ have non-vanishing boundary values on the UV brane and act as sources of corresponding CFT operators. In addition, due to the presence of the UV brane, the source fields obtain kinetic terms at quantum level and become dynamical. On the other hand, the Dirichlet boundary condition at the UV brane implies that $\psi^{(--)}$ and $\psi^{(-+)}$ do not take nonzero values on the UV brane and therefore they are not fundamental degrees of freedom in the UV regime of CFT.

The boundary conditions at the IR brane determine the low-energy description of CFT. The broken CFT generates various bound states. There are also composite states which have the same quantum number of $\psi^{(++)}$ and then only a combination of $\psi^{(++)}$ and these composite states remains massless. The mixture is determined by the anomalous dimension of corresponding CFT operator, in other words, a bulk mass of AdS field. On the other hand, $\psi^{(+-)}$ acquires a mass term with the corresponding massless CFT bound state $\mathcal{O}^{(+-)}$ and decouples at the IR scale. The state $\mathcal{O}^{(+-)}$ has an opposite quantum number to $\psi^{(+-)}$ and corresponds to a dynamical degree of freedom of $\psi^{(-+)}$ in the IR region. The field $\psi^{(--)}$ is understood as a massive bound state.

Now we can see how the anomaly is canceled in the IR. Since the massless modes at the IR scale are same as those in the effective four-dimensional theory in the AdS side, the anomaly cancels between these massless modes. In the UV region, $\psi^{(++)}$, $\psi^{(+-)}$, and fields living on the UV brane contribute to the $G$ gauge anomaly. There is also complicated CFT contribution to the anomaly and it is then nontrivial how these anomalies cancel out. However, due to the 't Hooft anomaly matching condition, the CFT-induced anomaly can be evaluated by massless composite states even for strongly-coupled CFT dynamics. The massless bound states are composed of $\mathcal{O}^{(+-)}$ and the fields which are dual to those living only on the IR brane. Since the anomaly from $\psi^{(+-)}$ is canceled by that from $\mathcal{O}^{(+-)}$, the conditions for anomaly cancellation become identical in the UV and IR regions of CFT [13]. According to the AdS/CFT correspondence, the CFT contribution to the $G$ anomaly is mapped to a Chern-Simons term in the AdS side. Therefore the action in the AdS side should have a Chern-Simons term whose coefficient is proportional to the sum of charges of $\mathcal{O}^{(+-)}$ and fermions localized on the IR brane. When we act a $G$ gauge transformation, the Chern-Simons term induces explicit $G$-breaking terms in the divergence of the current, which are localized on the two boundaries with equal magnitude and opposite sign. Since in the CFT dual theory the gauge symmetry $G$ is not broken, the breaking terms should be canceled out by fermion contributions and $G$ is not broken also in the AdS side. Thus we recover the localized anomaly in the case that the effective four-dimensional theory has a vector-like spectrum.

We have found that the five-dimensional gauge anomaly on the curved metric (2.3) is exactly the same as that in the flat case. It is independent of the metric factor $a(y)$ and bulk fermion masses. This result also follows from the theoretical arguments presented above. We thus conclude that orbifold theories in the non-factorizable geometries are free from anomalies when four-dimensional effective theories have anomaly-free spectra.

## 3. The Fayet-Iliopoulos term in warped geometry

In this section we study the Fayet-Iliopoulos (FI) terms in supersymmetric U(1) gauge theories on the $\mathrm{AdS}_5$ geometry. It is well known that a four-dimensional U(1) theory with charged matter fields has the one-loop FI term which is quadratically divergent and proportional to the sum of charges [14]. This radiatively-generated FI term is important for studying the vacuum of theory and it also has a deep connection to gravitational anomalies in supergravity theory. Therefore the characteristic features of FI terms in curved space-time deserve to be investigated similarly to the gauge anomalies examined in the previous section. The one-loop FI terms in flat five-dimensional orbifold theories were calculated in [7, 4] where they are induced via bulk and boundary charged fields. We investigate this phenomenon in U(1) gauge theories on the warped background. We will particularly find that the FI terms appear not only on the boundaries but also in the five-dimensional bulk. An interpretation of this is that the $\mathrm{AdS}_5$ supersymmetry requires different values of mass parameters for two chiral multiplets contained in one hypermultiplet.

Once the FI term is generated, it is a non-trivial problem whether one has a supersymmetric vacuum without breaking other symmetries. For example, a nonzero four-dimensional FI term necessarily causes supersymmetry and/or U(1) gauge symmetry breaking. However it has been shown that, in flat five-dimensional theories, both supersymmetry and gauge symmetry can survive in spite of the presence of FI terms, provided that the theory is free from U(1)-gravitational mixed anomaly [6, 15]. The analysis of flat directions shows that the scalar field in the U(1) vector multiplet develops a vacuum expectation value due to the FI term. This vacuum expectation value generates constant bulk masses for charged matter fields and significantly modifies the wavefunction profiles of KK modes [8] and [15]–[17], which have important consequences in low-energy effective theories. We will study a condition for supersymmetric vacuum and discuss phenomenological implications of the FI terms in the warped geometry.

### 3.1 Superspace action

We work with supersymmetric quantum electrodynamics in the warped (AdS) geometry. Its background metric is given by (2.3) with the metric factor $a(y) = e^{-k|y|}$. The fifth dimension $y$ has the two boundaries at $y = 0$ and $y = \pi R$ as in the previous section. To discuss radiative corrections to auxiliary scalar fields, it is relevant to use the off-shell superspace formalism of higher-dimensional supersymmetry [8, 18]. In the superfield language, the present theory involves two types of five-dimensional supermultiplets; vector and hyper multiplets. A five-dimensional off-shell vector multiplet contains a four-dimensional vector multiplet $V = (A_\mu,\, \lambda,\, D)$ and a neutral chiral multiplet $\chi = (\Sigma + iA_5,\, \lambda',\, F_\chi)$, where $\lambda$ and $\lambda'$ are gauge fermions, and $D$ and $F_\chi$ are auxiliary scalar fields. An off-shell hypermultiplet consists of oppositely-charged two chiral multiplets $\Phi = (\phi,\, \psi,\, F_\phi)$ and $\Phi^c = (\phi^c,\, \psi^c,\, F_{\phi^c})$. The five-dimensional action for AdS$_5$ supermultiplets is given in the $N = 1$ superspace form

$$\begin{aligned} S_V &= \int d^4x dy \left[ \left[\int d^2\theta \, \frac{1}{4g^2} W^\alpha W_\alpha + \text{h.c.} + \int d^2\theta d^2\bar\theta \, \frac{e^{-2k|y|}}{g^2} \left( \partial_y V - \frac{1}{\sqrt{2}}(\chi + \chi^\dagger) \right)^2 \right] \right], \\ S_H &= \int d^4x dy \left[ \left[ \int d^2\theta d^2\bar\theta \, e^{-2k|y|} \left( \Phi^\dagger e^{-qV} \Phi + \Phi^c e^{qV} \Phi^{c\dagger} \right) + \right. \right. \\ &\qquad \left. + \int d^2\theta \, e^{-3k|y|} \Phi^c \left[ \partial_y - \frac{q}{\sqrt{2}} \chi - \left( \frac{3}{2} - c \right) k\epsilon(y) \right] \Phi + \text{h.c.} \right], \end{aligned} \tag{3.1}$$

where $q$ and $c$ denote the charge and mass parameter of the chiral multiplet $\Phi$, and $\epsilon(y)$ is the sign function. The exponential warp factors have been included explicitly. For the lowest component of each supermultiplet, these warp factors describe the metric dependence such as $\sqrt{-g}$ in the AdS$_5$ action, but the metric factors for other component fields in the AdS$_5$ action are obtained after the following rescaling

$$\begin{aligned} &\lambda \to e^{-\frac{3}{2}k|y|}\lambda, \quad \lambda' \to e^{-\frac{1}{2}k|y|}\lambda', \quad D \to e^{-2k|y|}D, \quad F_\chi \to e^{-k|y|}F_\chi, \\ &\psi \to e^{-\frac{1}{2}k|y|}\psi, \quad \psi^c \to e^{-\frac{1}{2}k|y|}\psi^c, \quad F_\phi \to e^{-k|y|}F_\phi, \quad F_{\phi^c} \to e^{-k|y|}F_{\phi^c}. \end{aligned} \tag{3.2}$$

For later discussion, we mention that after integrating out the auxiliary fields, the scalar components $\phi$ and $\phi^c$ in a hypermultiplet have the following five-dimensional masses [19]

$$\begin{aligned} m_\phi^2 &= \left( c^2 + c - \frac{15}{4} \right) k^2 + (3 - 2c)k \left[ \delta(y) - \delta(y - \pi R) \right], \\ m_{\phi^c}^2 &= \left( c^2 - c - \frac{15}{4} \right) k^2 + (3 + 2c)k \left[ \delta(y) - \delta(y - \pi R) \right]. \end{aligned} \tag{3.3}$$

The boundary conditions imposed on the supermultiplets are similarly chosen as in section 2. The vector multiplet $V$ has the Neumann boundary conditions at both UV and IR branes and its superpartner multiplet $\chi$ has the Dirichlet ones because it contains the fifth component of bulk gauge field. Noting that a boundary condition of $\Phi^c$ must be opposite to that of superpartner $\Phi$ for respecting the $\mathbb{Z}_2$ orbifold, we have four different possibilities for boundary conditions of hypermultiplets. They are expressed as $(\Phi^{(++)},\, \Phi^{c\,(--)})$, $(\Phi^{(--)},\, \Phi^{c\,(++)})$, $(\Phi^{(+-)},\, \Phi^{c\,(-+)})$, and $(\Phi^{(-+)},\, \Phi^{c\,(+-)})$ with the notation introduced in the previous section. A hypermultiplet also carries two other parameters; its U(1) charge $q$ and bulk mass parameter $c$. It can be seen that the action is irrelevant under the replacement of $(\Phi^{(+\pm)},\, \Phi^{c\,(-\mp)})$ with $(q,\, c)$ by $(\Phi^{(-\mp)},\, \Phi^{c\,(+\pm)})$ with $(-q,\, -c)$. Therefore it is sufficient to show the results only for the two cases $\Phi^{(++)}$ and $\Phi^{(+-)}$ with generic values of $q$ and $c$. The results for the other parity assignments can be obtained by simply changing signs of charges and bulk mass parameters.

We also include the action for chiral multiplets confined on the UV and IR boundaries.

$$S_{\rm UV} = \int d^4x dy \left[\int d^2\theta d^2\bar\theta\, \Phi_{\rm UV}^\dagger e^{-q_{\rm UV}V}\Phi_{\rm UV} + \int d^2\theta\, W_{\rm UV}(\Phi, \Phi^c, \Phi_{\rm UV}) + {\rm h.c.}\right]\delta(y)\,, \tag{3.4}$$

$$\begin{aligned} S_{\rm IR} &= \int d^4x dy \left[\int d^2\theta d^2\bar\theta\, e^{-2k\pi R}\Phi_{\rm IR}^\dagger e^{-q_{\rm IR}V}\Phi_{\rm IR} + \right. \\ &\qquad \left. + \int d^2\theta\, e^{-3k\pi R} W_{\rm IR}(\Phi, \Phi^c, \Phi_{\rm IR}) + {\rm h.c.}\right]\delta(y-\pi R)\,. \end{aligned} \tag{3.5}$$

The warp factors have been taken into account in the IR brane action. The $\mathbb{Z}_2$ boundary conditions break a half of bulk supersymmetry and thus the boundary actions preserve only $N = 1$ supersymmetry. The boundary chiral multiplets $\Phi_{\rm UV}$ and $\Phi_{\rm IR}$ couple only to bulk multiplets with Neumann boundary conditions on each brane. We assume for simplicity that there are no $y$-derivative couplings of $\mathbb{Z}_2$-odd chiral multiplets and no four-dimensional gauge fields on the branes, though these assumptions are irrelevant to the following discussion.

For abelian gauge theory, the FI term of vector multiplet $V$ can also be added to the action

$$S_D = \int d^4x dy \int d^2\theta d^2\bar\theta\, 2\xi(y) V = \int d^4x dy\, \xi(y) D\,. \tag{3.6}$$

We have defined the coefficient $\xi(y)$ into which the metric warp factor is absorbed. When the fifth dimension is compactified in the orbifold, $\partial_y V - (\chi+\chi^\dagger)/\sqrt{2}$ becomes a supergauge invariant combination. The additional $(\chi + \chi^\dagger)/\sqrt{2}$ part is superfluous in flat theories but is not in curved theories. Such a term, however, takes no part in the following analysis of FI terms.

### 3.2 One-loop FI tadpoles

It is well known in four-dimensional abelian gauge theory that even if the FI term is set to be zero at the classical level, it is generated through loop-level divergent tadpole graphs. In the present five-dimensional case, bulk and boundary scalar fields contribute to the one-loop FI tadpoles. For bulk scalar fields, the relevant vertex is found from the action

$$-\frac{q}{2}e^{-2k|y|}\left(\phi^\dagger D\phi - \phi^c D\phi^{c\dagger}\right), \tag{3.7}$$

which induces a tadpole contribution to the auxiliary field $D$;

$$\xi(y) = -\frac{q}{2}e^{-2k|y|}\int\frac{d^4p}{(2\pi)^4}\left[G_p^\phi(y,y) - G_p^{\phi^c}(y,y)\right]. \tag{3.8}$$

The scalar propagators $G_p^{\phi,\,\phi^c}(y,y')$ on the $\mathrm{AdS}_5$ background are calculated in the mixed position/momentum space where $p$ is the four-dimensional momentum [20, 21]. Since the above superspace action already takes account of the metric factors for scalars, the Green's function evaluated at the coinciding points in the extra dimension is given by

$$G_p^{\phi}(y,y) = \frac{i\pi}{2k} e^{4k|y|} \left[ \widetilde{J}_{|c+\frac{1}{2}|}\left(\frac{ip}{k}\right) H_{|c+\frac{1}{2}|}\left(\frac{ip}{k}e^{k|y|}\right) - \widetilde{H}_{|c+\frac{1}{2}|}\left(\frac{ip}{k}\right) J_{|c+\frac{1}{2}|}\left(\frac{ip}{k}e^{k|y|}\right)\right] \times$$
$$\times \left[ \widetilde{J}_{|c+\frac{1}{2}|}\left(\frac{ip}{k}e^{k\pi R}\right) H_{|c+\frac{1}{2}|}\left(\frac{ip}{k}e^{k|y|}\right) - \widetilde{H}_{|c+\frac{1}{2}|}\left(\frac{ip}{k}e^{k\pi R}\right) J_{|c+\frac{1}{2}|}\left(\frac{ip}{k}e^{k|y|}\right)\right] \Big/$$
$$\Big/ \left[ \widetilde{J}_{|c+\frac{1}{2}|}\left(\frac{ip}{k}e^{k\pi R}\right) \widetilde{H}_{|c+\frac{1}{2}|}\left(\frac{ip}{k}\right) - \widetilde{H}_{|c+\frac{1}{2}|}\left(\frac{ip}{k}e^{k\pi R}\right) \widetilde{J}_{|c+\frac{1}{2}|}\left(\frac{ip}{k}\right)\right], \tag{3.9}$$

where $H_\alpha$ is the Hankel function of the first kind of order $\alpha$ and $J_\alpha$ is the Bessel function. The boundary condition of $\Phi$ at the UV (IR) brane determines the function forms of $\widetilde{J}$ and $\widetilde{H}$ with the argument $ip/k$ ($ipe^{k\pi R}/k$). If $\Phi$ has a Neumann boundary condition, then $\widetilde{J}_{|c+1/2|}(z) = \pm z J_{\pm(c-1/2)}(z)$ for $\pm(c+1/2) > 0$, while if a boundary condition is Dirichlet, $\widetilde{J}_{|c+1/2|}(z) = J_{|c+1/2|}(z)$, and similarly for $\widetilde{H}$. The propagator $G_p^{\phi^c}$ of the partner chiral multiplet of $\Phi$ can be obtained by replacement $c \to -c$ in all the above expressions.

Given the explicit expressions of Green's functions, we perform the four-dimensional momentum integral for the FI tadpole (3.8). The complicated but straightforward calculation shows that it is at most quadratically divergent and therefore we regulate it by introducing a sharp UV cutoff $\Lambda$. We then find the leading divergent FI term in the warped geometry, which is radiatively induced by a bulk hypermultiplet with charge $q$ and mass parameter $c$ :

$$\xi(y) = \frac{q}{32\pi^2}\left[\Lambda^2\left[\delta(y) + \eta\delta(y-\pi R)\right] + 2ck\Lambda\left[\delta(y) - e^{-k\pi R}\delta(y-\pi R)\right] - ck^2\Lambda e^{-ky} + \right.$$
$$\left. + \frac{1}{2}(\ln\Lambda)(\partial_y - k)\left[\delta'(y) + \eta e^{-2k\pi R}\delta'(y-\pi R)\right]\right]. \tag{3.10}$$

The parameter $\eta$ takes $+1$ for the boundary condition $\Phi^{(++)}$ and $-1$ for $\Phi^{(+-)}$. As we mentioned, the FI term from $\Phi^{(--)}$ $[\Phi^{(-+)}]$ can be derived by changing the signs of $q$ and $c$ in the result of $\Phi^{(++)}$ $[\Phi^{(+-)}]$. The net result of FI term is obtained by summing up all hypermultiplet contributions with different charges, masses, and boundary conditions. Note that the second line in the right-handed side of (3.10) does not correspond to a FI term but renormalizes higher-derivative brane-localized operators which are sub-leading in derivative-expansion approximation.

The four-dimensional boundary fields $\Phi_{\rm UV}$ and $\Phi_{\rm IR}$ also contribute the divergent FI term. It is easily found from the boundary actions (3.4) and (3.5) that one-loop tadpoles are calculated in the usual four-dimensional way and given by

$$\xi(y) = \frac{1}{16\pi^2}\Lambda^2\left[q_{\rm UV}\delta(y) + q_{\rm IR}\delta(y-\pi R)\right]. \tag{3.11}$$

The total amount of divergent FI terms is given by the sum of (3.10) and (3.11).

The FI term due to bulk hypermultiplets (3.10) shows that the quadratic divergences are localized at the boundaries and split evenly between the two sides. This is resemblance to the localized gauge anomalies found in the previous section. In fact, the quadratic divergence is proportional to the sum of matter charges, which is also proportional to the coefficient of mixed gravitational anomaly. The relation between FI terms and gravitational anomalies gives a deep understanding in supergravity embedding on the warped background. The embedding would also suggest that higher-order corrections do not induce additional FI terms. These issues are beyond the scope of this paper and we leave them to future investigations. Here we mention a relation between a possible anomaly cancellation and the structure of $\Lambda^2$ term. In the effective four-dimensional theory, a mixed gravitational anomaly vanishes if

$$\sum q_{++} - \sum q_{--} + \sum q_{\mathrm{UV}} + \sum q_{\mathrm{IR}} = 0 \,, \tag{3.12}$$

where $q_{++}$ and $q_{--}$ are the U(1) charges of $\Phi^{(++)}$ and $\Phi^{(--)}$, respectively. This anomaly-free condition leads to a total $\Lambda^2$ term of the form

$$\xi_{\Lambda^2}(y) = \frac{Q}{32\pi^2}\Lambda^2 \left[\delta(y) - \delta(y-\pi R)\right]. \tag{3.13}$$

The constant $Q$ is given by a relevant sum of charges: $Q = \sum q_{+-} - \sum q_{-+} + \sum q_{\mathrm{UV}} - \sum q_{\mathrm{IR}}$ with an obvious notation. One can see a kinship between gravitational anomalies and FI terms; the anomaly-free spectrum gives rise to a vanishing FI term in the effective four-dimensional theory. When the condition (3.12) is satisfied, the theory can be regularized with a set of Pauli-Villars fields with suitable charges and bulk masses such that no light mode with wrong statistics is left in the low-energy theory. Possible candidates for regulator hypermultiplets must only contain $\Phi^{(+-)}$ $[\Phi^{(-+)}]$ with bulk masses $c < \frac{1}{2}$ $(c > -\frac{1}{2})$. Other types of fields contain massless modes in the limit of infinitely large mass parameters.

The linearly divergent FI term comes only from hypermultiplets and represents characteristic properties due to the warped geometry. The suppression by the metric factor exists in the $\Lambda^1$ part. Furthermore, the $\Lambda^1$ FI term is not only confined on the boundaries but also penetrates into the five-dimensional bulk, which is quite different from the flat orbifold case where there is no FI term in the bulk. This behavior is also rather different from the $\Lambda^2$ term and the gauge anomalies. A conceivable understanding of this fact is based on the mass difference between two scalar fields in a hypermultiplet. On the AdS background, these two scalars have different graviphoton charges [22] and then have different masses in the bulk as well as on the branes. If one expands the integrand of the FI term (3.8) with respect to scalar masses, the first sub-leading order in terms of $p$ is proportional to the bulk mass difference. In fact, the bulk $\Lambda^1$ FI term agrees with this scalar mass difference. The additional exponential factors should come from the details of vertices and propagators. We also mention that in the limit $k \to 0$ with bulk masses $kc$ fixed, the radiative FI terms in flat theory [6] are properly recovered.

We finally comment on typical mass scales of radiative corrections on the boundaries. In the Randall-Sundrum model [1], the mass scale on the IR brane is suppressed by the exponential warp factor. This can be seen by use of the action after the rescaling (3.2). Then the $\Lambda^2$ divergence of the FI term at the IR brane becomes $(\Lambda e^{-k\pi R})^2$ and has a relatively suppressed mass scale.

## 4. Phenomenology of the FI terms

In the previous section, we observed that an anomaly-free condition in the low-energy theory makes the $\Lambda^2$ FI term of the form (3.13). In our field normalization, the zero mode of $D$, consisting of a four-dimensional vector multiplet with the zero mode of gauge field, has a flat wavefunction profile along the fifth dimension. Then the integrability condition for the FI coefficient

$$\int dy\, \xi(y) = 0 \tag{4.1}$$

is satisfied and we have a vanishing FI term in the effective four-dimensional theory. This is consistent with the fact that the FI term is proportional to the coefficient of U(1)-gravitational mixed anomaly in four dimensions, which is now zero.

The $D$-term equation is found from the five-dimensional supersymmetric action described before,

$$\begin{aligned} D = & -\partial_y(e^{-2k|y|}\Sigma) - g^2\xi(y) + \sum \frac{qg^2}{2} e^{-2k|y|}(\phi^\dagger\phi - \phi^{c\dagger}\phi^c) + \\ & + \sum \frac{q_{\rm UV} g^2}{2}\phi_{\rm UV}^\dagger \phi_{\rm UV}\delta(y) + \sum \frac{q_{\rm IR} g^2}{2} e^{-2k\pi R}\phi_{\rm IR}^\dagger\phi_{\rm IR}\delta(y-\pi R)\,, \end{aligned} \tag{4.2}$$

where $\Sigma$ is the neutral scalar in the U(1) vector multiplet. The supersymmetric condition $D = 0$ is governed by the first two terms if one neglects the flat directions of charged fields, some of which are lifted by other supersymmetric flatness conditions. As a result, the $D$-flatness condition becomes

$$\partial_y(e^{-2k|y|}\Sigma) + g^2\xi(y) = 0\,. \tag{4.3}$$

This equation has a solution only when the FI coefficient fulfills the integrability condition. Moreover, since $\Sigma$ is neutral, the vacuum does not break the U(1) gauge symmetry. The existence of such a vacuum preserving both supersymmetry and gauge symmetry is expected from a four-dimensional viewpoint, where there is no non-vanishing FI term which inevitably causes four-dimensional supersymmetry and/or gauge symmetry breaking.

### 4.1 Bulk field propagators

Keeping in mind the relation to anomalies, we now consider the FI coefficient of the form

$$\xi(y) = \frac{Q}{32\pi^2}\Lambda^2\left[\delta(y) - \delta(y-\pi R)\right] + \frac{Ck}{16\pi^2}\Lambda\left[\delta(y) - e^{-k\pi R}\delta(y-\pi R) - \frac{k}{2}e^{-ky}\right], \tag{4.4}$$

where $Q$ was defined in (3.13) and $C$ is the sum of $qc$ over all bulk hypermultiplets. We have simply dropped the logarithmically divergent term since they are less sensitive to the UV cutoff.[2] The solution to the $D$-flatness condition (4.3) is then given by

$$\Sigma(y) = -\frac{g^2}{64\pi^2}\epsilon(y)\left(Q\Lambda^2 e^{2k|y|} + 2Ck\Lambda e^{k|y|}\right). \tag{4.5}$$

[2] Including higher-derivative $\delta'$ and $\delta''$ terms would be important for more detailed analysis of KK-mode wavefunctions. Thank S. Groot Nibbelink for noticing us this issue. See also recent work [23].

The quadratically divergent part has a solution as we discussed. It is interesting that the linearly divergent part can also be compatible with a supersymmetric vacuum thanks to the existence of FI term spreading into the five-dimensional bulk (the last term in (4.4)). Given the vacuum expectation value of $\Sigma$, hypermultiplets acquire supersymmetry-preserving mass terms through the bulk superpotential. Since the value of $\Sigma$ is no longer constant in the extra dimension, so are the hypermultiplet masses. Such $y$-dependent masses could drastically change the KK-mode spectrum and KK phenomenology from that without FI terms or in flat orbifold theories. Note that boundary chiral multiplets do not receive any $D$-term contributions from FI terms. This is understood from the fact that $\Sigma$ obeys the Dirichlet boundary conditions at the orbifold fixed points.

Let us study the five-dimensional Green's functions of hypermultiplet scalars. The Green's functions for fermions behave in similar ways as long as supersymmetry is preserved. After integrating out the auxiliary components of bulk supermultiplets, we find the propagator of $\phi$ (with U(1) charge $q$ and a mass $c$) defined by

$$\left[e^{-2k|y|}\partial_\mu^2 + \partial_y(e^{-4k|y|}\partial_y) - e^{-4k|y|}M_\phi^2\right] G^\phi(x,x',y,y') = \delta^{(4)}(x-x')\delta(y-y')\,, \tag{4.6}$$

where the scalar mass-squared $M_\phi^2$ is

$$\begin{aligned} M_\phi^2 &= \bar{m}(y)^2 + e^{4k|y|}\partial_y\left[e^{-4k|y|}\bar{m}(y)\right], \\ \bar{m}(y) &\equiv \left(\frac{3}{2}-c\right)k\epsilon(y) + \frac{q}{2}\langle\Sigma(y)\rangle\,. \end{aligned} \tag{4.7}$$

The mass parameter of $M_{\phi^c}^2$ is given by the replacement $q\to -q$ and $c\to -c$. It is now convenient to work with the Green's function in the mixed position/momentum frame which is defined by Fourier-transforming with respect to the four-dimensional momentum

$$G^\phi(x,x',y,y') = \int\frac{d^4p}{(2\pi)^4}e^{ip(x-x')}G_p^\phi(y,y')\,. \tag{4.8}$$

Introducing the conformal coordinate $z=e^{ky}/k$ and $G_p^\phi(z,z')=\exp(\int_{z'}^z\frac{\bar{m}}{kw}dw)\widetilde{G}_p^\phi(z,z')$, the above equation becomes

$$\left[\partial_z^2 - 2\left(akz+b+\frac{c}{kz}\right)k\partial_z - p^2\right]\widetilde{G}_p^\phi(z,z') = (kz)^3\delta(z-z')\,, \tag{4.9}$$

where we have defined the two dimensionless parameters concerning the FI contributions; $a=\frac{qg^2}{128\pi^2k}Q\Lambda^2$ and $b=\frac{qg^2}{64\pi^2}C\Lambda$. We first consider the most dominant part of the FI term, i.e. $a\neq 0$ and $b=0$. The linear divergences do not cause meaningful changes in the conclusion we will show. In this case, the generic solution to the homogeneous equation is described by the Kummer's hypergeometric function. The solutions in two regions $z<z'$ and $z>z'$ must satisfy relevant boundary conditions at the fixed points and also the matching conditions at $z=z'$ (the continuity of $\widetilde{G}$ and a jumping condition for $\partial_z\widetilde{G}$). We thus find the general expression of Green's functions for arbitrary scalar fields

$$
\begin{aligned}
G_p^\phi(y,y') = &\frac{e^{4ky'}}{2kf(y')}\exp\left[\left(\frac{3}{2}-c\right)k(y-y')-\frac{a}{2}\left(e^{2ky}-e^{2ky'}\right)\right]\times\\
&\times\left[\widetilde{\mathcal{J}}_{c+\frac{1}{2}}\left(\frac{1}{k}\right)\mathcal{H}_{c+\frac{1}{2}}\left(\frac{e^{ky_<}}{k}\right)-\widetilde{\mathcal{H}}_{c+\frac{1}{2}}\left(\frac{1}{k}\right)\mathcal{J}_{c+\frac{1}{2}}\left(\frac{e^{ky_<}}{k}\right)\right]\times\\
&\times\left[\widetilde{\mathcal{J}}_{c+\frac{1}{2}}\left(\frac{e^{k\pi R}}{k}\right)\mathcal{H}_{c+\frac{1}{2}}\left(\frac{e^{ky_>}}{k}\right)-\widetilde{\mathcal{H}}_{c+\frac{1}{2}}\left(\frac{e^{k\pi R}}{k}\right)\mathcal{J}_{c+\frac{1}{2}}\left(\frac{e^{ky_>}}{k}\right)\right]\Big/\\
&\Big/\left[\widetilde{\mathcal{J}}_{c+\frac{1}{2}}\left(\frac{e^{k\pi R}}{k}\right)\widetilde{\mathcal{H}}_{c+\frac{1}{2}}\left(\frac{1}{k}\right)-\widetilde{\mathcal{H}}_{c+\frac{1}{2}}\left(\frac{e^{k\pi R}}{k}\right)\widetilde{\mathcal{J}}_{c+\frac{1}{2}}\left(\frac{1}{k}\right)\right], \qquad (4.10)
\end{aligned}
$$

where we have defined $y_<$ ($y_>$) to be the lesser (greater) of $y$ and $y'$. The functions $\mathcal{J}$ and $\mathcal{H}$ correspond to the two independent solutions to the homogeneous part of the equation (4.9) which are given by the Kummer's function

$$
\mathcal{J}_\beta(w) = (k\omega)^{2\beta}{}_1F_1\left(\frac{p^2}{4ak^2}+\beta;\,1+\beta;\,ak^2w^2\right), \quad \mathcal{H}_\beta(w) = {}_1F_1\left(\frac{p^2}{4ak^2};\,1-\beta;\,ak^2w^2\right). \qquad (4.11)
$$

In a special case that $\beta = n$ ($n$ : non-positive integer), the two functions are not independent (corresponding to $J_n$ and $J_{-n}$ in the flat limit), and $\mathcal{H}$ should be replaced by some independent function. We do not consider this special value of parameter but the analysis below does not lose any generalities. The boundary conditions on the propagator provide the forms of $\widetilde{\mathcal{J}}$ and $\widetilde{\mathcal{H}}$;

$$
\widetilde{\mathcal{J}}_\beta(w) = \begin{cases} \dfrac{\omega}{2}\mathcal{J}'_\beta(w) & \text{for Neumann} \quad ([\partial_y - \bar{m}(y)]G_p^\phi = 0) \\ \mathcal{J}_\beta(w) & \text{for Dirichlet} \quad (G_p^\phi = 0) \end{cases} \qquad (4.12)
$$

and similarly for $\widetilde{\mathcal{H}}_\beta$. For example, if $\phi$ is free on the UV (IR) brane, then $\widetilde{\mathcal{J}}_\beta(\frac{1}{k}) = \frac{1}{2k}\mathcal{J}'_\beta(\frac{1}{k})$ $[\widetilde{\mathcal{J}}_\beta(\frac{e^{k\pi R}}{k}) = \frac{e^{k\pi R}}{2k}\mathcal{J}'_\beta(\frac{e^{k\pi R}}{k})]$. With these functions, $f(y)$ is written as

$$
f(y) = \frac{e^{ky}}{2k}\left[\mathcal{J}'_{c+\frac{1}{2}}\left(\frac{e^{ky}}{k}\right)\mathcal{H}_{c+\frac{1}{2}}\left(\frac{e^{ky}}{k}\right)-\mathcal{H}'_{c+\frac{1}{2}}\left(\frac{e^{ky}}{k}\right)\mathcal{J}_{c+\frac{1}{2}}\left(\frac{e^{ky}}{k}\right)\right]. \qquad (4.13)
$$

In the limit of vanishing FI term ($a \to 0$), the dimensionless function $f(y)$ behaves as $e^{(2c+1)ky}$.

### 4.2 KK spectrum and wavefunction profiles

#### 4.2.1 $\Phi^{(++)}$ and $\Phi^{(--)}$ : a massless mode

The four-dimensional mass spectrum of chiral supermultiplets are extracted from pole conditions of the five-dimensional propagator (4.10). The conditions crucially depend on the boundary conditions of chiral multiplets. The general form of pole conditions becomes

$$
\widetilde{\mathcal{J}}_{c+\frac{1}{2}}\left(\frac{e^{k\pi R}}{k}\right)\widetilde{\mathcal{H}}_{c+\frac{1}{2}}\left(\frac{1}{k}\right)-\widetilde{\mathcal{H}}_{c+\frac{1}{2}}\left(\frac{e^{k\pi R}}{k}\right)\widetilde{\mathcal{J}}_{c+\frac{1}{2}}\left(\frac{1}{k}\right)=0\,. \qquad (4.14)
$$

In this equation, the four-momentum has been replaced by mass eigenvalue $p^2 = -m^2$. For a $\Phi^{(++)}$ multiplet, the equation is explicitly given by

$$\frac{m^2}{4ak^2}\Bigg[e^{(2c-1)k\pi R}{}_1F_1\left(\frac{-m^2}{4ak^2}+c+\frac{1}{2};\,c+\frac{1}{2};ae^{2k\pi R}\right){}_1F_1\left(\frac{-m^2}{4ak^2}+1;\frac{3}{2}-c;a\right)- \\ -{}_1F_1\left(\frac{-m^2}{4ak^2}+c+\frac{1}{2};\,c+\frac{1}{2};\,a\right){}_1F_1\left(\frac{-m^2}{4ak^2}+1;\,\frac{3}{2}-c;\,ae^{2k\pi R}\right)\Bigg]=0\,. \tag{4.15}$$

It is clearly seen that there is always a massless mode. Since the present vacuum does not break the low-energy supersymmetry, a fermionic partner of this scalar zero mode is also massless and they make up a massless chiral multiplet in four dimensions. On the other hand, the equation (4.14) for $\Phi^{(--)}$ turns out to become

$$e^{(2c+1)k\pi R}{}_1F_1\left(\frac{-m^2}{4ak^2}+c+\frac{1}{2};\,c+\frac{3}{2};\,ae^{2k\pi R}\right){}_1F_1\left(\frac{-m^2}{4ak^2};\,\frac{1}{2}-c;a\right)- \\ -{}_1F_1\left(\frac{-m^2}{4ak^2}+c+\frac{1}{2};c+\frac{3}{2};\,a\right){}_1F_1\left(\frac{-m^2}{4ak^2};\,\frac{1}{2}-c;ae^{2k\pi R}\right)=0\,. \tag{4.16}$$

The eigenvalue equations for a $\Phi^c$ chiral multiplet are obtained by replacing $a \to -a$ and $c \to -c$. Using a formula for the Kummer's function ${}_1F_1(\alpha;\beta;\gamma) = e^{\gamma}{}_1F_1(\beta-\alpha;\beta;-\gamma)$, it can be checked that the pole condition of $\Phi^{c\,(--)}$ is identical to that of the KK-excited modes of $\Phi^{(++)}$. Therefore the KK massive spectrum of $\Phi^{c\,(--)}$ is paired up with that of $\Phi^{(++)}$, leaving one four-dimensional massless chiral multiplet in $\Phi^{(++)}$. The KK-excited modes have a complicated form of mass spectrum depending on the signs of FI term and bulk mass parameter, but in the case of a large FI term, the KK mass eigenvalues start from around $a^{1/2}$, not suppressed by the exponential warp factor. We also explicitly checked that a vanishing FI term recovers the mass eigenvalue equations in the Randall-Sundrum background derived in [21].

Let us examine the wavefunction profiles of KK chiral multiplets. It is easily found that all the massive KK modes are localized to the IR brane for any values of U(1) charges and bulk mass parameters. These excited modes become rather heavy and may have no significant effects on the low-energy theory, and therefore we will focus on the massless eigenstate in $\Phi^{(++)}$. The detailed study of KK wavefunctions, including the effects of higher-derivative terms [the second line in (3.10)], will be presented elsewhere. The equation of motion for the massless mode $\phi_0(x)\chi_0^{(++)}(y)$ is given by

$$\partial_y\left[e^{-4k|y|}\partial_y(\chi_0^{(++)})\right]-e^{-4k|y|}M_\phi^2\chi_0^{(++)}=0\,. \tag{4.17}$$

The mass-squared $M_\phi^2$ was defined by Eq. (4.7). Solving the equation with the Neumann boundary conditions at the fixed points, we obtain

$$\chi_0^{(++)}(y)=N_0\exp\left[\left(\frac{3}{2}-c\right)k|y|-\frac{a}{2}e^{2k|y|}\right]. \tag{4.18}$$

The constant $N_0$ is fixed by the normalization $\int dye^{-2k|y|}|\chi_0^{(++)}|^2 = 1$ so that the four-dimensional massless mode $\phi_0(x)$ has a canonical kinetic term. The normalization condition means that $N_0$ contains a factor $e^{\frac{a}{2}}$ for $a > 0$ (a factor $e^{\frac{a}{2}e^{2k\pi R}}$ for $a < 0$), which ensures the flat limit, $k \to 0$ with $ck$ and $ak$ fixed, reproduces the zero-mode wavefunction found in the literature. If one considers the boundary conditions $(\Phi^{(--)}, \Phi^{c(++)})$ and has a massless mode in $\Phi^c$, one should reverse the signs of $q$ and $c$ in the solution.

As seen from the above expression, the wavefunction profile of the zero mode highly depends on $a$. Keeping in mind a warp factor $e^{-2k|y|}$ in the kinetic term of hypermultiplet, we have four different cases depending on the values of two parameters $a$ and $c$.

(i) $a > 0,\ c > \frac{1}{2}$ and (ii) $a < 0,\ c < \frac{1}{2}$

In these cases, the zero modes have monotonously varying wavefunctions. They have peaks at the boundary $y = 0$ ($y = \pi R$) for the case (i) [(ii)].

(iii) $a > 0,\ c < \frac{1}{2}$ and (iv) $a < 0,\ c > \frac{1}{2}$

In these cases, the zero-mode wavefunctions have the extreme values between the two branes, which are the maximum and the minimum for the cases (iii) and (iv), respectively. It is also found that, in the case (iii) [(iv)], the value of the wavefunction at $y = 0$ is always larger (smaller) than that at $y = \pi R$. The peaks of the zero modes are located at the position

$$y = \frac{1}{2k} \ln \left( \frac{\frac{1}{2} - c}{a} \right), \tag{4.19}$$

which can take various values between the UV and IR branes, depending on a relative size of $a$ and $c$. If $|a|$ is very large, the wavefunction profile is similar to the case (i) or (ii). The situation is changed for a relatively small value of $|a|$, where the FI-term coefficient is around the scale on the IR brane. In this case, we find for the case (iii) that the massless chiral multiplet is peaked at a middle point between the two branes. On the other hand, for the case (iv), the massless mode is localized on the both boundaries $y = 0$ and $y = \pi R$. These characteristic profiles may provide a novel approach to four-dimensional phenomenology.

#### 4.2.2 $\Phi^{(+-)}$ and $\Phi^{(-+)}$ : a pair of almost massless modes

For the other types of boundary conditions $\Phi^{(+-)}$ and $\Phi^{(-+)}$, the pole conditions are also extracted from the general expression of Green's functions. For a $\Phi^{(+-)}$ chiral multiplet, the condition becomes

$$\begin{aligned} &\left(c^2 - \frac{1}{4}\right) e^{-(2c+1)k\pi R} {}_1F_1\left(\frac{-m^2}{4ak^2}; \frac{1}{2} - c; ae^{2k\pi R}\right) {}_1F_1\left(\frac{-m^2}{4ak^2} + c + \frac{1}{2}; c + \frac{1}{2}; a\right) - \\ &\quad - \left(\frac{m^2}{4k^2}\right) {}_1F_1\left(\frac{-m^2}{4ak^2} + 1; \frac{3}{2} - c; a\right) {}_1F_1\left(\frac{-m^2}{4ak^2} + c + \frac{1}{2}; c + \frac{3}{2}; ae^{2k\pi R}\right) = 0, \end{aligned} \tag{4.20}$$

and for $\Phi^{(-+)}$,

$$\begin{aligned} &\left(c^2 - \frac{1}{4}\right) e^{(2c-1)k\pi R} {}_1F_1\left(\frac{-m^2}{4ak^2} + c + \frac{1}{2}; c + \frac{1}{2}; ae^{2k\pi R}\right) {}_1F_1\left(\frac{-m^2}{4ak^2}; \frac{1}{2} - c; a\right) - \\ &\quad - \left(\frac{m^2}{4k^2}\right) {}_1F_1\left(\frac{-m^2}{4ak^2} + c + \frac{1}{2}; c + \frac{3}{2}; a\right) {}_1F_1\left(\frac{-m^2}{4ak^2} + 1; \frac{3}{2} - c; ae^{2k\pi R}\right) = 0. \end{aligned} \tag{4.21}$$

One can easily find from these equations that there are no exactly massless modes. They have been projected out by the boundary conditions from the KK spectrum. The pole condition for $\Phi^{c\,(+-)}$ ($\Phi^{c\,(-+)}$) is given by replacement $a \to -a$ and $c \to -c$ in eq. (4.20) ((4.21)) and is equivalent to the pole condition of $\Phi^{(-+)}$ ($\Phi^{(+-)}$). Therefore the KK modes from $\Phi^{(+-)}$ and $\Phi^{c\,(-+)}$ ($\Phi^{(-+)}$ and $\Phi^{c\,(+-)}$) necessarily come in pair. Though almost all of the KK modes become heavy due to the presence of FI term ($a \neq 0$), there can be one pair of light chiral multiplets in the low-energy effective theory. The mass of these light modes is explicitly derived by expanding the eigenvalue equation (4.20) (or (4.21)) with a large FI term

$$m^2 \simeq 4a^2k^2 \exp\left[(1-2c)k\pi R - a(e^{2k\pi R}-1) + \frac{\pi R m^2}{ak}\right]. \tag{4.22}$$

If $a > 0$, the last factor in the right-handed side can be dropped and we have

$$m \simeq 2ak \exp\left[\left(\frac{1}{2}-c\right)k\pi R - \frac{a}{2}(e^{2k\pi R}-1)\right]. \tag{4.23}$$

Our derivation shows that such light modes appear in two types of hypermultiplets; ($\Phi^{(+-)}$, $\Phi^{c\,(-+)}$) with a positive charge $a > 0$ and ($\Phi^{(-+)}, \Phi^{c\,(+-)}$) with a negative charge $a < 0$. The wavefunctions of the light chiral multiplets are shown to have the following form

$$\begin{aligned} \chi_l^{(+-)} &\simeq N_l \exp\left[\left(\frac{3}{2}-c\right)k|y| - \frac{a}{2}e^{2k|y|}\right], \\ \chi_l^{c\,(-+)} &\simeq N_l^c \exp\left[\left(\frac{3}{2}+c\right)k|y| + \frac{a}{2}e^{2k|y|}\right], \end{aligned} \tag{4.24}$$

for a positive $a$. The normalization constants $N_l$ and $N_l^c$ are determined by $\int dy e^{-2k|y|} \times |\chi_l^{(+-)}|^2 = 1$, etc. Note that the wavefunctions of $\chi_l^{(-+)}$ and $\chi_l^{c\,(+-)}$ with a negative $a$ are exactly the same as the above, but in the expression of their mass eigenvalue the signs of $a$ and $c$ must be changed from (4.23). The $\chi_l^{(+-)}$ wavefunction is similar to the massless mode $\chi_0^{(++)}$ derived in (4.18) where U(1) charge parameter $a$ was free. This similarity is because the mass (4.23) is negligibly small and $\chi_l^{(+-)}$ and $\chi_0^{(++)}$ satisfy almost the same equation of motion. As seen from the above expressions, the eigenfunction is suppressed at the boundary where the Dirichlet boundary condition is imposed and has a large value at the opposite boundary.

We have found, for any values of charges and bulk masses, there are two light chiral multiplets with a tiny mass between these two. For a large FI term, $\chi_l^{(+-)}$ and $\chi_l^{c\,(-+)}$ are strongly localized, with little overlap, onto the UV and IR branes, respectively. The resultant spectrum for ($\Phi^{(+-)}, \Phi^{c\,(-+)}$) or ($\Phi^{(-+)}, \Phi^{c\,(+-)}$) is like the supersymmetric quantum chromodynamics though we have imposed the chiral $\mathbb{Z}_2$ projection on the spectrum.

If one takes the flat background limit ($k \to 0$ with $ck \equiv \bar{c}$ and $ak \equiv \bar{a}$ fixed), the tiny mass eigenvalue (4.23) becomes

$$m \simeq 2\bar{a}e^{-(\bar{a}+\bar{c})\pi R}. \tag{4.25}$$

This corresponds to an exponentially-suppressed KK mass found in ref. [16] for a vanishing bulk mass parameter ($\bar{c} = 0$).



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### 4.3 Effective theory and physical implications

In the previous section, we found three types of (almost) massless chiral multiplets in the low-energy effective theory. $\Phi^{(++)}$ has a massless mode independent of $a$, but its wavefunction highly depends on $a$. With a positive (negative) $a$ parameter, we have an almost massless vector-like modes in $\Phi^{(+-)}$ and $\Phi^{c\,(-+)}$ ($\Phi^{(-+)}$ and $\Phi^{c\,(+-)}$). The multiplets $\Phi^{(++)}$, $\Phi^{(+-)}$ with $a>0$ and $\Phi^{c\,(+-)}$ with $a<0$ are localized at the UV brane and the others are at the IR brane. As for a U(1) neutral multiplet with $(++)$ boundary condition, it gives a massless chiral multiplet and the wavefunction is controlled by the bulk mass parameter $c$. In addition, there is a four-dimensional massless vector multiplet $V_0$ whose wavefunction is constant along the extra dimension. We schematically describe low-energy effective theories for these light modes as well as four-dimensional boundary chiral multiplets with the following action

$$S^{\rm eff}_{\rm vector} = \int d^4x \int d^2\theta\, \frac{1}{4g_4^2} W_0^\alpha W_{0\alpha} + \text{h.c.}\,, \tag{4.26}$$

$$\begin{aligned} S^{\rm eff}_{\rm chiral} = \int d^4x \int d^2\theta d^2\bar\theta \Big[ & \Phi_0^\dagger e^{-q_0 V_0}\Phi_0 + \Phi_l^\dagger e^{-q_l V_0}\Phi_l + \Phi_l^c e^{q_l V_0}\Phi_l^{c\,\dagger} + \\ & + K^{\rm eff}_{\rm UV}\Big(\Phi_{\rm UV}, \Phi_0^{a>0}, \chi_0^{(++)}(0)\Phi_0^{a<0}, \Phi_l\Big) + \\ & + e^{-2k\pi R} K^{\rm eff}_{\rm IR}\Big(\Phi_{\rm IR}, \chi_0^{(++)}(\pi R)\Phi_0^{a>0}, \Phi_0^{a<0}, \Phi_l^c\Big)\Big] + \\ + \int d^4x \int d^2\theta \Big[ & W^{\rm eff}_{\rm UV}\Big(\Phi_{\rm UV}, \Phi_0^{a>0}, \chi_0^{(++)}(0)\Phi_0^{a<0}, \Phi_l\Big) + \\ & + e^{-3k\pi R} W^{\rm eff}_{\rm IR}\Big(\Phi_{\rm IR}, \chi_0^{(++)}(\pi R)\Phi_0^{a>0}, \Phi_0^{a<0}, \Phi_l^c\Big)\Big] + \text{h.c.}\,, \end{aligned} \tag{4.27}$$

where $g_4$ is the gauge coupling defined by $1/g_4^2 = \pi R/g^2$. We have not explicitly written down $O(1)$ factors of wavefunctions for notational simplicity. The four-dimensional field $\Phi_0(x)$ denotes a light degree of freedom from $\Phi^{(++)}$, and $\Phi_l(x)$ and $\Phi_l^c(x)$ come from the multiplets with boundary conditions $(+-)$ and $(-+)$ and are localized at the UV and IR branes, respectively. In the boundary Kähler potentials and superpotentials, only the multiplets with Neumann boundary conditions appear with relevant strengths on each brane. The extension to multi flavors of hypermultiplets is straightforward. The Kähler potentials $K^{\rm eff}_{\rm UV}$ and $K^{\rm eff}_{\rm IR}$ contain the kinetic terms for boundary multiplets $\Phi_{\rm UV}$ and $\Phi_{\rm IR}$, and also include possible higher-dimensional operators among bulk and brane superfields. The superpotentials $W_{\rm UV}$ and $W_{\rm IR}$ preserve only half of bulk supersymmetry and they can involve mass, Yukawa, and other terms allowed in four-dimensional supersymmetric theory. The superspace form is relevant to study various phenomenology in the low-energy effective theory preserving supersymmetry.

The above effective action shows that the vector-like multiplets $\Phi_l$ and $\Phi_l^c$ behave in a similar way to boundary chiral multiplets as long as their charge parameters $a$'s are not so small. On the other hand, $\Phi_0$ multiplets appear with the factors of wavefunctions evaluated at the boundaries. The wavefunction factors appeared in (4.27) are

$$\chi_0^{(++)}(0) \;\simeq\; \sqrt{\frac{2ke^{-a}}{\ln(-a)}}\,\lambda^{\frac{1}{2}(c-\frac{3}{2})}\,e^{+\frac{a}{2\lambda}} \qquad (a<0)\,, \tag{4.28}$$

$$\chi_0^{(++)}(\pi R) \simeq \sqrt{\frac{2ke^{a}}{\ln a}}\,\lambda^{\frac{1}{2}(c-\frac{3}{2})}\,e^{-\frac{a}{2\lambda}} \qquad (a>0)\,. \tag{4.29}$$

We have defined a small parameter $\lambda = e^{-2k\pi R}$. These wavefunction factors give sources to generate two different sizes of hierarchies, which are given by powers and exponentials of the ratio between the UV and IR scales. Taking various values of charges and bulk masses, we obtain hierarchical quantities in the low-energy effective theory. This provides a new tool for low-energy phenomenology.

As the first example, we discuss a large hierarchy among the Yukawa couplings of quarks and leptons. While the masses of quarks and charged leptons are in the vicinity of the electroweak scale, the recent experimental observations indicate that neutrinos have tiny masses of $O(\mathrm{eV})$, which are $10^{-(11-14)}$ times smaller than the electroweak scale. Our aim in the current example is to explain such a huge hierarchy between the masses of neutrinos and other fermions by exploiting the suppression factors of (4.28) and (4.29). As an illustration, we assume that the electroweak Higgs fields and Yukawa terms are confined on the IR brane. In addition, the quarks and charged leptons come from the zero modes of bulk hypermultiplets with $(++)$ boundary conditions, mass parameters $c_i$, and vanishing U(1) charges.[3] In this case, the bulk masses $c_i$ determine the wavefunction factors of charged fermions at the IR brane $\chi_0^{++}(\pi R)$ and hence generate Yukawa hierarchies between the three generations. This is archived when the radius modulus is stabilized so that $\lambda \sim 10^{-(1-2)}$. We also introduce right-handed neutrino multiplets with positive $a$ parameters. (The U(1) gauge invariance requires at least one extra standard-model singlet (or a doublet Higgs) with a negative $a$ parameter.) The non-vanishing $a$ induces a large suppression of neutrino wavefunctions at the IR brane and the tiny neutrino Yukawa couplings follow. It is found from (4.29) that the suppression is roughly given by the exponential factor $e^{-a/2\lambda}$ which can give a correct order of magnitude for neutrino masses with an $O(1)$ value of $a$. We also note that a mild hierarchy between the three neutrino masses is realized by choosing different neutrino bulk masses $c_{\nu_i}$.

Another application is concerned with the Planck/weak mass hierarchy. We now want to show a simultaneous realization both of Yukawa and Planck/weak mass hierarchies, where the former is generated by the difference of wavefunctions caused by bulk matter masses and therefore the parameter $\lambda$ is set to be about a unit of Yukawa hierarchy. The Plank/weak mass hierarchy is then realized by U(1) charge parameter $a$. As a primitive example, let us consider that five-dimensional boundary superpotentials include the following terms for Higgs fields

$$W_{\mathrm{UV}} = \frac{f_S}{\sqrt{M}}S(x,y)H_u(x)H_d(x)\,, \qquad W_{\mathrm{IR}} = M^{3/2}S(x,y)\,, \tag{4.30}$$

[3] In fact, it is not necessarily to place the entire matter multiplets in the five-dimensional bulk. A simple way to produce the right fermion masses is to only put a set of 10 representations of SU(5) in additional spatial dimensions [24].

where $f_S$ is a dimensionless coupling and $M$ denotes the fundamental scale of five-dimensional theory, which is not much different from the Plank scale. We have assumed that the doublet Higgses $H_u$ and $H_d$ are confined on the UV brane and the standard-model singlet $S$ comes from a bulk hypermultiplet with (++) boundary condition. Now $S$ and $H_d$ have opposite charge parameters $a$ and $-a$, respectively ($a > 0$) so that the dimension-four term $W_{\rm UV}$ is gauge invariant.[4] One-loop chiral anomalies can be canceled, e.g. by adding appropriate charged multiplets to boundary theories. Given the charge assignment, $S$ contains a zero mode localized at the UV brane. It is found from (4.27) that the supersymmetric vacuum in four dimensions satisfies

$$\langle H_u\rangle\langle H_d\rangle = \frac{-1}{f_S}\left(\frac{\chi_s(\pi R)}{\chi_s(0)}\right) e^{-3k\pi R} M^2 \,, \tag{4.31}$$

where $\chi_s$ denotes the wavefunction of the zero mode of $S$ whose form is read from (4.29). Such a wavefunction factor of $S$ appears because the massless mode needs to propagate the bulk space to communicate the coupling at the IR brane to the UV brane. Supersymmetry is not broken though we have a linear superpotential in $W_{\rm IR}$. This is understood by solving the $F$-flat conditions explicitly and also from the four-dimensional effective action. For example, a vanishing bulk mass parameter of the $S$ field leads to the vacuum expectation values of the doublet Higgses

$$\langle H_u\rangle,\, \langle H_d\rangle \simeq \lambda^{3/8} e^{-\frac{a}{4\lambda}} M \,, \tag{4.32}$$

The parameter $\lambda$ defined before gives a unit of Yukawa hierarchy. Eq. (4.32) thus shows that the Planck/weak mass hierarchy is obtained by choosing $O(1)$ $a$ parameter.

The Yukawa couplings of quarks and leptons come from the UV boundary term

$$W_{\rm UV} = \frac{1}{M} f_u^{ij} Q_i u_j H_u + \frac{1}{M} f_d^{ij} Q_i d_j H_d + \frac{1}{M} f_l^{ij} L_i e_j H_d \tag{4.33}$$

with the standard notation. All the matter multiplets are assumed to reside in the bulk and the couplings $f_{u,d,l}$ are dimensionless $O(1)$ parameters. For these terms to be gauge invariant, the U(1) charges of quark and lepton multiplets are $a_Q = a_u = a_e = 0$ and $a_d = a_L = +a$ ($> 0$). For simplicity we assume $\mathrm{U}(1)_Y$ of the standard model gauge group does not have FI term. The zero modes of $d$ and $L$ are then localized at the UV brane and do not provide any suppressions of Yukawa couplings $f_d$ and $f_l$. We take the mass parameters of U(1)-neutral multiplets $c_{Q,u,e} < \frac{1}{2}$ so that they are peaked at the IR brane (see the localization property discussed in section 4.2.1). As a result, the zero-modes quarks and leptons are found to have the following Yukawa couplings

$$(f_u^{ij})_0 \;\simeq\; \frac{\lambda^{1-c_{Q_i}-c_{u_j}}}{\pi M R}\,, \qquad (f_d^{ij})_0 \;\simeq\; \frac{\lambda^{\frac{1}{2}-c_{Q_i}}}{\pi M R}\,, \qquad (f_l^{ij})_0 \;\simeq\; \frac{\lambda^{\frac{1}{2}-c_{e_j}}}{\pi M R}\,. \tag{4.34}$$

Several interesting results follow from this expression. First, the hierarchy of up-quark masses is generally larger than that of down quarks due to the suppression effect of the right-handed up quarks, while not disturbing the smallness of quark mixing angles. On

[4] The U(1) symmetry is softly broken by $W_{\rm IR}$. A potential alternative is $\mathrm{U}(1)_R$ symmetry under which $S$ has charge +2 and both $W_{\rm UV,\,IR}$ are invariant. In this case, we are necessarily lead to supergravity theory.

the other hand, the hierarchy of lepton Yukawa couplings are controlled only by the right-handed electrons. This implies that the left-handed leptons can largely mix with each other, which is indeed suggested by the recent experimental results for neutrino physics. If one takes the bulk masses $c_Q = c_u = c_e$ motivated by grand unification, the effective Yukawa couplings (4.34) predict the mass eigenvalues and mixing angles roughly consistent with the present experimental data. We comment that $W_{\rm UV}$ in (4.30) could also lead to a natural $\mu$-term generation by introducing additional terms of $S$ on the IR brane.

The mechanism proposed above is different from other approaches to the hierarchy problems with the AdS warp factor. In refs. [21, 25], the Higgs fields (and Yukawa couplings) are confined on the TeV brane to have the electroweak scale from the warp factor à la Randall-Sundrum [1]. The matter multiplets have suitable bulk masses $c_i$ and are localized at the Planck brane. The situation generates Yukawa hierarchy in unit of a ratio between the Planck/weak mass hierarchy, namely, $f_{ij} \sim (M_{\rm weak}/M_{\rm Pl})^{c_i+c_j-1}$, and therefore needs some fine-tuning of mass parameters $c_i$ for realistic fermion masses and mixing.

## 5. Conclusions and discussions

In this paper we have discussed chiral gauge anomalies in curved spacetime and also the structure of Fayet-Iliopoulos terms in warped supersymmetric theories. The chiral anomalies have been found to appear at the orbifold fixed points and are localized in the exactly same fashion as in flat theories. This result has the theoretical supports from a low-energy effective theory viewpoint and also from the AdS/CFT correspondence. The gauge anomaly is canceled by an appropriate Chern-Simons term if the four-dimensional effective theory is free from gauge anomaly.

Unlike the gauge anomaly, the FI terms behave differently from the flat theory. In the warped geometry, the FI terms are generated not only on the boundaries but also in the whole bulk. When the effective theory is free from gravitational anomalies, there is a supersymmetric vacuum without breaking the gauge symmetry in spite of non-vanishing FI term. This consistent behavior is in fact guaranteed by the presence of the bulk FI term. In turn, the effect of FI term becomes to generate supersymmetric masses for charged bulk multiplets which depend on the metric factor. Contrary to the Randall-Sundrum model where the scale of KK masses is suppressed, typical KK masses for charged particles are around the scale of FI coefficient. The wavefunction profiles of bulk multiplets have also been examined for various types of boundary conditions on the branes. For example, the massless mode in $\Phi^{(++)}$ with even-even parity is strongly localized onto the UV or IR brane, depending on its U(1) charge and bulk mass parameter. We have shown by explicit constructions that this localization behavior has interesting applications to the problems in particle physics, e.g., Yukawa and Planck/weak mass hierarchies.

It may be interesting to study the localized anomaly and FI terms via the four-dimensional theory space approach [26]. The field theory in the AdS$_5$ spacetime can also be formulated along this line [27] and it should be possible to examine gauge anomalies as in [28]. The structure of FI terms is also understood in a similar way.



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In this paper we have not considered supersymmetry breaking and gravity. The localized FI terms and resultant modified wavefunctions significantly affect supersymmetry breaking in the low-energy theory. Several ways to break supersymmetry are expected in the AdS background, e.g., orbifolding, brane-localized interactions, the radius modulus $F$ term. Each mechanism could lead to distinguishable sparticle spectrum in the low-energy effective theory. Combined with other phenomenological issues, realistic model construction along this line may deserve to be investigated. Proceeding to supergravity theory, we could study more clearly the relations between FI terms and gravitational anomalies. The invariance under local supersymmetry might suggest a possible form of the coefficient of FI term. To examine gravitational back-reactions due to FI terms may also be an interesting issue.

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## Addendum

In this paper we introduced a sharp UV cutoff $\Lambda$ in four-dimensional momentum integral. Since the fifth direction is curved, this cutoff may depend on its coordinate $y$. If we work with $\Lambda(y) = \Lambda \exp(-ky)$ as motivated by AdS/CFT correspondence, we obtain the results consistent with supergravity calculation. In this case, we have supersymmetric vacua with nonzero expectation values of charged matter fields. For example the phenomenological implications of FI term discussed in section 4 are completely unchanged when IR boundary chiral multiplets are introduced. If UV boundary fields instead of IR ones get vacuum expectation values, we then need to replace $\Lambda$ by $\Lambda \exp(-k\pi R)$ in the analysis of section 4.